

All of these problems count as bonus problems, i.e. they do not count towards the maximum possible homework problems, but if you solve them they count towards your score.

1. **Epistemic Logic:** Draw an epistemic state for two actors,  $A$  and  $B$ , where:

- $p$  and  $q$  are both true in the actual world
- $A$  believes  $p$  to be false, and does not know whether  $q$  is true or false
- $B$  believes both  $p$  and  $q$  to be false
- Each of the two actors knows what the other one knows

2. **Epistemic Logic:** Given the epistemic state shown in Figure 1, for each of the following sentences, determine if the state is a model for that sentence (and why). Note that left and right refer to the left and right card, respectively, which may either be clubs ( $\clubsuit$ ) or spades ( $\spadesuit$ )

- $\Box_A \text{ left}(\clubsuit)$
- $\Box_A \text{ left}(\spadesuit)$
- $\Box_B \text{ right}(\clubsuit)$
- $\Box_B \text{ right}(\spadesuit)$
- $\Box_B (\text{left}(\spadesuit) \vee \text{left}(\clubsuit))$
- $(\Box_B \text{ left}(\spadesuit)) \vee (\Box_B \text{ left}(\clubsuit))$

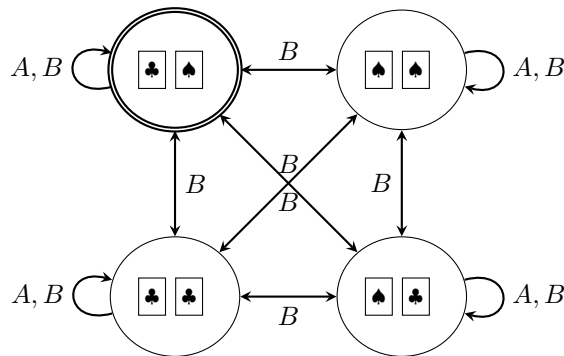


Figure 1: An Epistemic State.

3. **Epistemic Logic:** Given the epistemic state shown in Figure 1, for each of the following sentences, determine if the state is a model for that sentence (and why). Note that left and right refer to the left and right card, respectively, which may either be clubs ( $\clubsuit$ ) or spades ( $\spadesuit$ )

- $\Box_B \Box_A \text{left}(\clubsuit)$
- $\Box_A \Box_A \text{left}(\spadesuit)$
- $\Box_B (\text{right}(\clubsuit) \rightarrow \Box_A \text{right}(\clubsuit))$
- $\Box_A (\Box_B \text{left}(\spadesuit)) \vee (\Box_B \text{left}(\clubsuit))$
- $\Box_A \Box_B \Box_A \text{right}(\spadesuit)$

4. **Dynamic Epistemic Logic:** Apply the epistemic action  $(? \text{left}(\clubsuit))^*B$  to the epistemic state in figure 1 and draw the resulting epistemic state.

5. **Dynamic Epistemic Logic:** Write an epistemic action that has the following effect: The left card is replaced with a  $\spadesuit$ ,  $A$  knows that the card has been replaced, but does not know if it now is a  $\spadesuit$  or a  $\clubsuit$ , and  $B$ 's beliefs stay unchanged. Apply that action to the epistemic state in figure 1 and draw the resulting epistemic state.